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MULTIVALUED FIXED POINT THEOREM IN NON ARCHIMEDEAN VECTOR SPACE

Amita Joshi
Department of Mathematics, IPS Academy, Indore, India

ABSTRACT
In this paper we extend Theorem A for two mappings.

Keywords- *Non-Archimedean normed space, multivalued contractive mapping.*

I. INTRODUCTION

Let (X, ||.||)be a non-Archimedean normed space (for definition see [5]). We say that (X, ||.||) is spherically complete if every shrinking collection of balls in X has a non-empty intersection.

We denote by C(X) the family of all non-empty compact subsets of X, by CB(X) the family of all non-empty closed bounded subsets of X, and by H the Hausdorff metric on CB(X) induced by d ; that is
$H(A,B) = \max\{\sup\{d(a,B) ; a \in A\} , \sup\{d(b,A) ; b \in B \}$
for all A, B in CB(X) , where $d(x,A) = \inf\{d(x,y) ; y \in A\}$ for all x in X and $A \subset X$.
$T : X \to CB(X)$ is said to be multivalued contractive (non-expansive) mapping if
$H(Tx,Ty) < \|x - y\|$ for any distinct points in X.
($H(Tx,Ty) \le \|x - y\|$ for any $x,y \in X$)

It is known that a contractive mapping in a complete metric space need not to have a fixed point see for examples ([1],[4]).

Petalas and Vidalis [3] have proved that in a non-Archimedean spherically complete normed space (X, ||.||) every contractive mapping has a unique fixed point.

II. MAIN RESULTS

Kubiaczyk and Ali [2] extended this result for multivalued mappings and proved the following :
TheoremA . Let X be a non-Archimedean spherically complete normed space. If $T : X \to C(X)$ is a mapping such that $H(Tx,Ty) < \|x - y\|$ for any two distinct points x and y in X. Then T has a fixed point.

In this chapter we extend Theorem A for two mappings. We prove the following:

Theorem 1 . Let X be a non-Archimedean spherically complete normed space. If $T_1, T_2 : X \to CB(X)$ is a mapping such that
(1) $H(T_1x,T_2y) < \|x - y\|$,
for any distinct points $x,y \in X$. Then T_1 and T_2 have a common fixed point in X. i.e. there exists $z \in X$ such that $z \in T_1z$ and $z \in T_2z$.

Proof . Let $B_a = B(a, d(a, T_1a) \wedge d(a, T_2a))$ denote the closed sphere centered a with the radius $r = \min \{d(a, T_1a),d(a, T_2a)\}$ and let
$A = \{B_a ; a \in X \}$ be the collection of all these spheres for all $a \in X$.
The relation

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$$B_a \leq B_b \text{ iff } B_b \subseteq B_a,$$

is a partial order . Consider a totally ordered subfamily A_1 of A. From the spherical completeness of X, we have

$$\bigcap_{B_a \in A_1} B_a = B \neq \phi$$

Let $b \in B$ and $B_a \in A_1$. Using $B_b \subseteq B_a$, we have

$$\|a - b\| \leq d(a, T_1a) \wedge d(a, T_2a)$$

Then if $x \in B_b$,

$$\|x - b\| \leq d(b, T_1b) \wedge d(b, T_2b)$$
$$\leq \max \{ \|b - a\| , \|a - d\| , \inf_{c \in T_2b} \|d - c\| \}$$

where $d \in T_1a$ be such that

$$\|a - d\| = d(a, T_1a) + \in \text{ (if for example } \{d(a, T_1a) \wedge d(a, T_2a)\} = d(a, T_1a)\text{, then}$$
$$\|x - b\| \leq \max\{d(a, T_1a) \wedge d(a, T_2a), d(d, T_2b) \} + \in$$
$$\leq \max\{d(a, T_1a) \wedge d(a, T_2a), H(T_1a, T_2b) \} + \in$$
$$\leq [d(a, T_1a) \wedge d(a, T_2a)] + \in.$$

Hence

$$\|x - a\| \leq \max \{ \|x - b\| , \|b - a\| \}$$
$$\leq [d(a, T_1a) \wedge d(a, T_2a)] + \in.$$

Since $\in$ is arbitrary, so making $\in \rightarrow 0$, these means

$$\|x - a\| \leq [d(a, T_1a) \wedge d(a, T_2a)] .$$

So $x \in B_a$ and $B_b \subseteq B_a$, for every $B_a \in A_1$.

By Zorn's lemma , A has a maximal element say Bz, for some $z \in B$.
We claim that $z \in T_1z$ and T_2z.
Let $z \neq T_1z \cap T_2z$ and $\check{z} \in T_1z \cup T_2z$ be such that $\check{z} \in T_2z$

(2) $$\|z - \check{z}\| \leq \min \{d(z, T_1z), d(z, T_2z)\} + \in.$$

Now we shall show that $B\check{z} \subseteq Bz$.
If $y \in B\check{z}$ then $\|\check{z} - y\| \leq d(\check{z}, T_1\check{z}) \wedge d(\check{z}, T_2\check{z}) \leq d(\check{z}, T_1\check{z})$

(3) $$\leq H(T_2z, T_1\check{z}) < d(z, \check{z})$$
$$< [d(z, T_1z) \wedge d(z, T_2z)] + \in.$$

By (2) and (3), we have

$$\|y - z\| \leq \min \{ \|y - \check{z}\| , \|\check{z} - z\| \}$$
$$\leq [d(z, T_1z) \wedge d(z, T_2z)] + \in.$$

Because this inequality hold for any $\in$, so making $\in \rightarrow 0$, we get

$$\|y - z\| \leq d(z, T_1z) \wedge d(z, T_2z) ,$$

This means that $y \in Bz$ so $B\check{z} \subseteq Bz$.
On the other hand

$$\|\check{z} - y\| > H(T_2z, T_1\check{z}) \geq d(\check{z}, T_1\check{z})$$
$$\geq \min\{d(\check{z}, T_1\check{z}) \wedge d(\check{z}, T_2\check{z})\}.$$

Hence $z \notin B\check{z}$, but $z \in Bz$. Therefore $B\check{z} \subseteq Bz$, and this contradicts the maximality of Bz.
Thus T_1 and T_2 have a common fixed point.
This completes the proof of the theorem.

III. CONCUSION

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Remark 1 . If the mappings T_1 and T_2 are functions. Theorem 1 state that the pair of functions such that, with condition (1) (not necessary continuous) has a common fixed point z.

Remark 2 . If we put $T_1 = T_2 = T$ in Theorem 1, we obtain result due to Kubiaczyk and Ali [2].

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